Polarization and topological charge conversion of exact optical vortex beams at normal incidence on planar dielectric interfaces

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We report on the exact resolution of the problem of reflection and refraction of exact circularly polarized Bessel vortex beams impinging at normal incidence on a planar dielectric interface between two isotropic and lossless media. On the one hand, we demonstrate the generation of a new vortex state both in the reflected and refracted fields. On the other hand, we show the possibility to completely convert, at reflection, the incident vortex beam into a vortex beam with orthogonal polarization and topological charge changed by ±2. The spin–orbit interaction of light occurring at the planar interface is identified as the mechanism responsible for these effects.

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Optical vortex beams (OVBs), which are light beams endowed with phase singularities [1], are useful in communications, imaging, and micromanipulation [2]. The behavior of OVBs at dielectric interfaces is a basic consideration when light–matter interaction is considered. In particular, the reflection and refraction of OVBs on planar interfaces has been extensively studied in the framework of the paraxial approximation. Subtle effects that involve both the spin and orbital contribution of the angular momentum of light actually occur at oblique incidence [3,4]. At normal incidence, a common belief is that both the polarization and topological charge (in the laboratory frame) are preserved for both the reflected and refracted fields [1]. However, this is true only in the paraxial limit where the cross-polarization coupling, which leads to generation of new states, is known to be negligible for paraxial Laguerre–Gaussian (LG) and Hermite–Gaussian (HG) beams [5–8]. Besides, until now, such effects have been analyzed in the beam spatial domain only numerically [5–7].

Here we report on the exact analytical spatial domain analysis of circularly polarized Bessel vortex beams (CBVBs) of arbitrary topological charge—a particular set of exact solutions of the Maxwell's equations—at normal incidence on a planar dielectric interface between two homogeneous, isotropic, and lossless media. We show the generation of a new vortex state both in transmitted and reflected fields. Moreover, we demonstrate the possibility of complete conversion at reflection of an incident CBVB into a new vortex state with orthogonal polarization and topological charge changed by ±2. Such an efficient conversion results from the combination of nonparaxiality and the specific angular spectrum structure of Bessel beams (BBs). Therefore, the full conversion is a unique property of Bessel vortices that can not be achieved with LG and HG beams. Moreover, we identify the optical spin–orbit interaction occurring at the planar interface as the mechanism responsible for the reported effects.

CBVBs of arbitrary topological charge l propagating in the z direction in the cylindrical coordinate system (r, φ, z) with orthonormal basis $(\mathbf{e}_r, \mathbf{e}_\varphi, \mathbf{e}_z)$ are described by the following electric and magnetic fields:

$$\begin{aligned}\mathbf{E} &= [J_l(\alpha r)\mathbf{e}_r + i\sigma J_l(\alpha r)\mathbf{e}_\varphi \\ &\quad - i\sigma \tan\theta_q J_{l+\sigma}(\alpha r)\mathbf{e}_z]e^{i[(l+\sigma)\varphi+\beta_q z]}, \\ \mathbf{H} &= \{-i\sigma[\cos\theta_q J_l(\alpha r) + \frac{l+\sigma}{k_q r}\tan\theta_q J_{l+\sigma}(\alpha r)]\mathbf{e}_r \\ &\quad + [\cos\theta_q J_l(\alpha r) + \sigma\tan\theta_q \sin\theta_q J'_{l+\sigma}(\alpha r)]\mathbf{e}_\varphi \\ &\quad - \sin\theta_q J_{l+\sigma}(\alpha r)\mathbf{e}_z\}e^{i[(l+\sigma)\varphi+\beta_q z]}. \end{aligned} \tag{1}$$

Here, $\sigma = \pm 1$ stands for circular polarization states, J_l is the lth order Bessel function of the first kind, $J'_l(x) = dJ_l/dx$, $\alpha = k_q \sin\theta_q$ and $\beta_q = k_q \cos\theta_q$ are, respectively, the transverse and longitudinal wave numbers with $k_q = k_0 n_q$, k_0 the wave number in vacuum and n_q the refractive index where the CBVB propagates. In addition, the time-dependent factor $e^{-i\omega t}$ is omitted. The name of such a beam is chosen on purpose since (i) it belongs to the family of nondiffracting BBs [9], (ii) its polarization state defined in the transverse plane is uniformly circular and (iii) its field possesses a global phase factor of the form $e^{il\varphi}$. Finally, let us recall that a BB can be conceived of as an evenly distributed superposition of plane waves whose wave vectors lie on a cone, making an angle $\theta_q = \arctan(\alpha/\beta_q)$ with the beam propagation axis.

The problem of CBVB reflection and refraction at normal incidence on a planar dielectric interface is treated by considering the plane $z = 0$ as the interface between two dielectric media with refractive index n_i for $z < 0$ and n_t for $z > 0$. In that case, since the TM-polarized and TE-polarized BBs [10] of arbitrary order m preserve their spatial structure, we will use them for our analysis. Following the results in [11–13], TM-polarized and TE-polarized BBs of order m can be expressed as

$$\begin{aligned}
\mathbf{E}_{\mathrm{TM}}^{(q)} &= \tau_p^{(q)}[-\cos\theta_q \boldsymbol{\xi}_\perp + i\sin\theta_q J_m(\alpha r)\mathbf{e}_z]e^{i(m\varphi+\beta_q z)}, \\
\mathbf{H}_{\mathrm{TM}}^{(q)} &= \tau_p^{(q)} n_q \boldsymbol{\eta}_\perp e^{i(m\varphi+\beta_q z)}, \\
\mathbf{E}_{\mathrm{TE}}^{(q)} &= \tau_s^{(q)} \boldsymbol{\eta}_\perp e^{i(m\varphi+\beta_q z)}, \\
\mathbf{H}_{\mathrm{TE}}^{(q)} &= \tau_s^{(q)} n_q[\cos\theta_q \boldsymbol{\xi}_\perp - i\sin\theta_q J_m(\alpha r)\mathbf{e}_z]e^{i(m\varphi+\beta_q z)}, \qquad (2)
\end{aligned}$$

where $q = (i, r, t)$ stands for the incident, reflected, and transmitted fields, respectively, and the transverse vectors $\boldsymbol{\xi}_\perp$ and $\boldsymbol{\eta}_\perp$ are defined as

$$\begin{aligned}
\boldsymbol{\xi}_\perp &= J_m'(\alpha r)\mathbf{e}_r + i\frac{m}{\alpha r}J_m(\alpha r)\mathbf{e}_\varphi, \\
\boldsymbol{\eta}_\perp &= i\frac{m}{\alpha r}J_m(\alpha r)\mathbf{e}_r - J_m'(\alpha r)\mathbf{e}_\varphi, \qquad (3)
\end{aligned}$$

where the expression of the fields parameters that depend on the index q are summarized in Table 1.

Noticeably, one can recognize (i) the expressions of θ_r and θ_t as the Snell's law that governs the reflection and refraction angles for plane waves and (ii) the expressions of $\tau_p^{(q)}$ and $\tau_s^{(q)}$ as the Fresnel equations for p and s polarized plane waves [14]. Such an analogy is explained by considering the angular spectrum representation of a BB that involves a single angle of incidence for all the constituting partial plane waves.

The incident CBVB state $|\psi\rangle^{(i)} \equiv |\sigma, l, i\rangle$ given by Eq. (1) is represented in terms of the $|\mathrm{TM}_m^{(i)}\rangle$ and $|\mathrm{TE}_m^{(i)}\rangle$ states as

$$|\psi\rangle^{(i)} = -i|\mathrm{TE}_{l+\sigma}^{(i)}\rangle - \frac{\sigma}{\cos\theta_i}|\mathrm{TM}_{l+\sigma}^{(i)}\rangle. \qquad (4)$$

Then, the reflected and transmitted states $|\psi\rangle^{(q)}$ are obtained from Eq. (4), accounting for the behavior of the TM and TE states at the interface,

$$|\psi\rangle^{(q)} = -i|\mathrm{TE}_{l+\sigma}^{(q)}\rangle - \frac{\sigma}{\cos\theta_i}|\mathrm{TM}_{l+\sigma}^{(q)}\rangle, \qquad (5)$$

where the reflected and transmitted the TM and TE states can be expressed as linear superposition of two distinct CBVBs. Namely, by using Eqs. (1) and (2), it can be shown that

Table 1. Parameters for the TM-polarized and TE-Polarized BBs

q	i	r	t
θ_q	θ_i	$\pi - \theta_i$	$\arcsin(\frac{n_i}{n_t}\sin\theta_i)$
β_q	$k_i\cos\theta_i$	$-\beta_i$	$\beta_i\frac{n_t\cos\theta_t}{n_i\cos\theta_i}$
$\tau_p^{(q)}$	1	$\frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)}$	$\frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}$
$\tau_s^{(q)}$	1	$-\frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)}$	$\frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)}$

$$\begin{aligned}
|\mathrm{TM}_m^{(q)}\rangle &= \frac{\tau_p^{(q)}}{2}\cos\theta_q[|-1, m+1, q\rangle - |1, m-1, q\rangle], \\
|\mathrm{TE}_m^{(q)}\rangle &= i\frac{\tau_s^{(q)}}{2}[|-1, m+1, q\rangle + |1, m-1, q\rangle]. \qquad (6)
\end{aligned}$$

Combining Eq. (5) and Eq. (6) we obtain

$$|\psi\rangle^{(q)} = c_1^{(q)}|\sigma, l, q\rangle + c_2^{(q)}|-\sigma, l+2\sigma, q\rangle, \qquad (7)$$

where the coefficients $c_{1,2}^{(q)}$ are given by

$$\begin{aligned}
c_{1,2}^{(t)} &= \frac{1}{2}\left[\tau_s^{(t)} \pm \tau_p^{(t)}\frac{\cos\theta_t}{\cos\theta_i}\right], \\
c_{1,2}^{(r)} &= \frac{1}{2}\left[\tau_s^{(r)} \mp \tau_p^{(r)}\right]. \qquad (8)
\end{aligned}$$

The analysis is completed by examining how the energy of the incident beam is divided between the two secondary beams. For this purpose, we define the transmittance $T = \lim_{\rho\to\infty}\frac{\mathcal{J}_z^{(t)}}{\mathcal{J}_z^{(i)}}$ and reflectance $R = \lim_{\rho\to\infty}\left|\frac{\mathcal{J}_z^{(r)}}{\mathcal{J}_z^{(i)}}\right|$ of the incident CBVB, where $\mathcal{J}_z^{(q)} = \frac{c}{8\pi}\mathrm{Re}\{\iint_{\mathcal{D}}[(\mathbf{E}^{(q)} \times \mathbf{H}^{(q)*})\cdot\mathbf{e}_z]\mathrm{d}x\mathrm{d}y\}$ is the time-averaged energy flux of the beam through a disk $\mathcal{D}$ of radius ρ lying in the $z = 0$ plane and centered on the origin; c is the speed of light in vacuum, and Re and the asterisk refer to real part and complex conjugation, respectively. After calculations we obtain:

$$T = \frac{(\tau_p^{(t)})^2 + (\tau_s^{(t)})^2\cos^2\theta_i}{1+\cos^2\theta_i}\,\frac{n_t\cos\theta_t}{n_i\cos\theta_i}, \qquad (9)$$

$$R = \frac{(\tau_p^{(r)})^2 + (\tau_s^{(r)})^2\cos^2\theta_i}{1+\cos^2\theta_i}. \qquad (10)$$

Equation (7) emphasizes the generation of a new vortex state due to the presence of the interface. The transmitted and reflected fields appear as the superposition of two distinct CBVBs: (i) $|\sigma, l, q\rangle$, with the same polarization and topological charge (in the laboratory frame) as the incident vortex field and (ii) $|-\sigma, l+2\sigma, q\rangle$, with orthogonal polarization and a topological charge that differs by 2σ. This allows us to identify the optical spin—orbit interaction occurring at the planar interface as the mechanism responsible for the effect that belongs to the broad class of spin-to-orbital angular momentum conversion phenomena that take place in focusing, scattering, and imaging systems [15].

The dependence of the reflection and refraction of an incident CBVB $|\sigma, l, i\rangle$ on the characteristic cone angle θ_i are displayed in Fig. 1 for two typical situations that are a refractive index ratio $N = n_t/n_i = 3/2$ [panels on the left] and $N = n_t/n_i = 2/3$ [panels on the right].

As shown on the first two rows, the creation of the vortex states $|-\sigma, l+2\sigma, t\rangle$ and $|-\sigma, l+2\sigma, r\rangle$ embedded in the transmitted [panels (a) and (d)] and reflected [panels (b) and (e)] fields, respectively, occur whatever $\theta_i > 0$ for which $c_2^{(r)} \neq 0$ and $c_2^{(t)} \neq 0$. The process is efficient, however, for sufficiently large incident

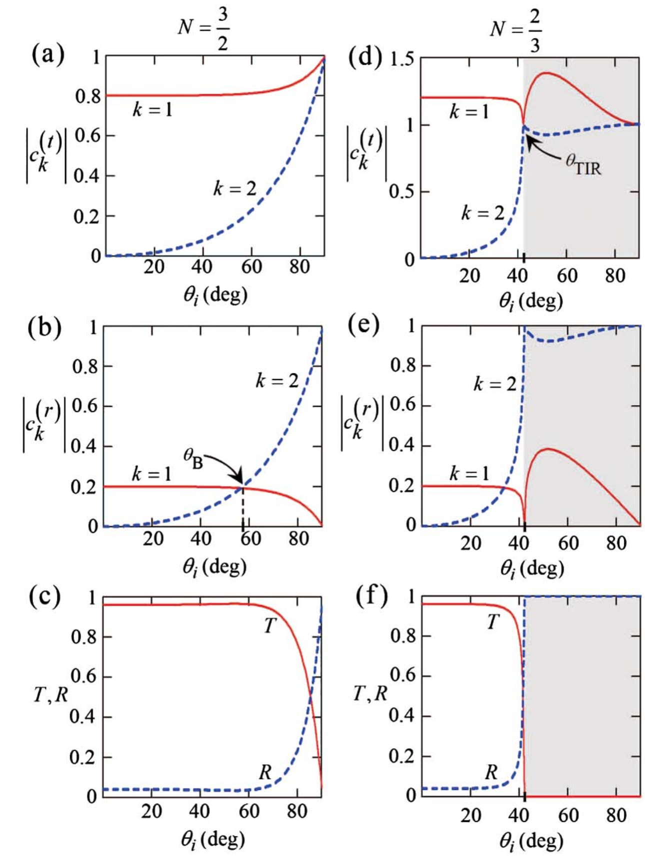

Fig. 1. (Color online) Dependence of the reflection and refraction of an incident CBVB on the characteristic cone angle θ_i for a refractive index ratio $N = n_t/n_i = 3/2$ [panels on the left] and $N = 2/3$ [panels on the right]. First and second rows: $c_k^{(\tau)}$ for the transmitted [panels (a) and (d)] and reflected [panel (b) and (e)] beams. Third row: transmittance T and reflectance R. The shadowed area on the right column refers to the total internal reflection regime when $\theta_i > \theta_{\mathrm{TIR}}$.

cone angles, which emphasizes the role played by nonparaxiality.

Still, the incident $|\sigma, l, i\rangle$ can be completely converted into the reflected one $|-\sigma, l + 2\sigma, r\rangle$. This occurs for $\theta_i = \theta_{\mathrm{TIR}}$, where θ_{TIR} is the total internal reflection angle [above which $T = 0$ and $R = 1$, as shown in Fig. 1(f)] defined by $\sin \theta_{\mathrm{TIR}} = N$ when $N < 1$. As seen in Fig. 1(e), at the critical angle we have

$$c_1^{(r)}(\theta_{\mathrm{TIR}}) = 0, \qquad c_2^{(r)}(\theta_{\mathrm{TIR}}) = 1. \tag{11}$$

Note that, in the spectral domain, Eq. (11) corresponds to the change of the handedness for each circularly polarized plane wave components of a CBVB. This explains why the full conversion can not be achieved for LG and HG beams. In addition, Eq. (11) points out the key role of the interface characteristics since full conversion at reflection is obtained for relatively low cone angles for small enough N.

Another particular situation corresponds to $\theta_i = \theta_B$, θ_B being the Brewster angle defined by $\tan \theta_B = N$ when $N > 1$. In that case $\tau_p^{(r)} = 0$ and $c_1^{(r)} = c_2^{(r)}$, as shown in Fig. 1(b), thereby leading to $|\psi\rangle^{(r)} = -i|\mathrm{TE}_{l+\sigma}^{(r)}\rangle$. This offers an alternative option to the recently introduced method to generate an azimuthally polarized field from the reflection at normal incidence on a planar interface of a zeroth-order BB whose cone angle matches the Brewster angle [16]. In our case, an azimuthal polarization state is obtained from the reflection of an incident CBVB with topological charge $l = -\sigma$: $\mathbf{E}^{(r)} = -i\tau_s^{(r)} J_1(\alpha r) e^{i\beta_r z} \mathbf{e}_\varphi$.

Finally, we note that the CBVB described by Eq. (1) identifies with the formulation of nonparaxial vortex beams given by Eq. (3.3) in [17] when using $E(\kappa) = \delta(\kappa - \alpha)$ for the spectral function. Consequently, our analysis can be readily generalized toward the exact spatial domain treatment of the reflection and refraction of arbitrary optical vortices at normal incidence on planar interfaces.

To conclude, the behavior of exact circularly polarized Bessel vortex beams impinging at normal incidence onto a planar isotropic dielectric interface has been described. The reflection and refraction of such optical vortices are accompanied with the generation of a new vortex state whose topological charge changes by ± 2, depending on the handedness of the incident circular polarization state as a result of the spin–orbit interaction of light mediated by the interface. In particular, we show that (i) total conversion of the incident vortex into a distinct one and (ii) generation of an azimuthally polarized beam can be achieved in the reflection mode.

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